Lie Brackets and State Constraints in Optimal Control Problems

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Abstract

The inward pointing condition (IPC) for a control system whose state x is constrained in a smooth set C requires that at each point of the boundary of C the intersection between the dynamics and the interior of the tangent cone of C at x be nonempty. Thanks to IPC, for every system trajectory $\bar{x}(\cdot)$ on an interval $[0, T]$, possibly violating the constraint by an amount d, one can construct a new system trajectory $x(\cdot)$ that both satisfies the constraint and whose distance from $\bar{x}(\cdot)$ is bounded by a quantity proportional to d. When IPC does not hold, the construction of such a trajectory is not possible in general. This paper is devoted to a state constrained control problem that is affine and symmetric with respect to the control and possibly contains an uncontrolled drift, that is not too large with respect to the controllable vector fields (in a sence that can be specified). We formulate an inward pointing condition involving Lie brackets of the dynamics' vector fields and, by implementing a suitable "rotating" control strategy, we construct a constrained trajectory whose distance from the reference trajectory $\bar{x}$ is bounded by a quantity proportional to $\sqrt{d}$. As an application, we establish the continuity up to the boundary of the value function of related optimal control problems with finite and infinite horizon. The main result removes two assumptions that were used in the preceding papers [B, C] and allows to prove, in the case of a driftless dynamics, that the mere inward pointing conditions involving the Lie bracket is enough to obtain the continuity of value functions. The presentation is based on the paper [A].

A Colombo, G., Rampazzo, F., Shishmintsev, D., *Neighboring feasible trajectories and value functions for state constrained control problems*, ESAIM Control Optim. Calc. Var. 31 (2025), Paper No. 83, 34 pp.

B Colombo, G., Khalil, N., Rampazzo, F,. *State constraints, higher order inward pointing conditions, and neighboring feasible trajectories*, SIAM J. Control Optim Vol. 60 (2022), 3326–3357.

C Colombo, G., Rampazzo, F., Vinter R. B., *Discontinuous Solutions to the Hamilton Jacobi Equation under Second Order Interiority Hypotheses*, Math. Contr. Rel. Fields Vol. 14 (2024), 1428–1437.